Forming and Solving Equations (H)

Intervention Booklet

Name: _____

Class: _____

Useful websites:

www.mathswatchvle.com
(Video explanations and questions)
Username: STH…@twgash
Password: stmaths

www.methodmaths.com
(Past papers online that get instantly marked)
Centre ID: wga
Username: firstname
Password: lastname

www.hegartymaths.com
(Online tutorials and quizzes)
Login: first name and last name are case sensitive

www.bbc.co.uk/schools/gcsebitesize/maths

Iterations

Things to remember:
- Make sure you know how to rearrange formulae first.
- Parts (b) and (c) usually involve substitution and occasionally solving.
- To prove a solution lies between two values, substitute them both in and look for a change of sign.

Questions:

1.   (a)   Show that the equation x³ + 4x = 1 has a solution between x = 0 and x = 1.

(2)

       (b)   Show that the equation x³ + 4x = 1 can be rearranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

(1)

       (c)   Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate for the solution of x³ + 4x = 1.

(3)
(6 marks)

2. An approximate solution to an equation is found using this iterative process:

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \quad \text{and} \quad x_1 = 1$$

(a) Work out the values of x_2 and x_3

x_2 = ………………………..

x_3 = ………………………..

(2)

(b) Work out the solution to 6 decimal places.

………………………………………………..

(1)

(3 marks)

3. (a) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

(2)

(b) Using
$$x_{n+1} = 3 + \frac{3}{x_n{}^2} \qquad \text{with } x_0 = 3.2$$
find the values of x_1, x_2 and x_3.

....
(3)

(c) Explain what the values of x_1, x_2 and x_3 represent.

....

....

....
(1)
(6 marks)

Expand and Factorise Quadratics

Things to remember:
- Use FOIL (first, outside, inside, last) or the grid method (for multiplication) to expand brackets.
- For any quadratic $ax^2 + bx + c = 0$, find a pair of numbers with a sum of b and a product of ac to factorise.

Questions:

1. Expand and simplify $(m + 7)(m + 3)$

....

(Total for question = 2 marks)

2. (a) Factorise $\quad 6 + 9x$

....

(1)

(b) Factorise $\quad y^2 - 16$

....

(1)

(c) Factorise $\quad 2p^2 - p - 10$

....

(2)

(Total for Question is 4 marks)

3. Solve, by factorising, the equation $\quad 8x^2 - 30x - 27 = 0$

....

(Total for Question is 3 marks)

4. Factorise $x^2 + 3x - 4$

....
(Total for question is 2 marks)

5. Write $x^2 + 2x - 8$ in the form $(x + m)^2 + n$ where m and n are integers.

....
(Total for question is 2 marks)

6. (a) Expand $4(3x + 5)$

....
(1)

(b) Expand and simplify $2(x - 4) + 3(x + 5)$

....
(2)

(c) Expand and simplify $(x + 4)(x + 6)$

....
(2)
(Total for Question is 5 marks)

7. (a) Factorise $x^2 + 5x + 4$

....
(2)

(b) Expand and simplify $(3x - 1)(2x + 5)$

....
(2)
(Total for Question is 4 marks)

8. (a) Expand $3(2 + t)$

....
(1)

(b) Expand $3x(2x + 5)$

....
(2)

(c) Expand and simplify $(m + 3)(m + 10)$

....
(2)
(Total for Question is 5 marks)

9. (a) Factorise $x^2 + 7x$

....
(1)

(b) Factorise $y^2 - 10y + 16$

....
(2)

*(c) (i) Factorise $2t^2 + 5t + 2$

....

(ii) t is a positive whole number.
The expression $2t^2 + 5t + 2$ can never have a value that is a prime number.
Explain why.

....

....

....
(3)
(Total for Question is 6 marks)

Using the Quadratic Formula

Things to remember:
- For any quadratic, $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Questions:

1. Solve $3x^2 + 7x - 13 = 0$
Give your solutions correct to 2 decimal places.

x = ... or x = ...

(3 marks)

2. Solve the equation
$2x^2 + 6x - 95 = 0$
Give your solutions correct to 3 significant figures.

x = ... or x = ...

(3 marks)

3. Solve $x^2 + 3x - 5 = 0$
Give your solutions correct to 4 significant figures.

....

(3 marks)

4. Solve this quadratic equation.
$$x^2 - 5x - 8 = 0$$
Give your answers correct to 3 significant figures.

x = ...or x = ...

(3 marks)

5. (a) Solve $x^2 - 2x - 1 = 0$
Give your solutions correct to 2 decimal places.

....

(3)

(b) Write down the solutions, correct to 2 decimal places, of $3x^2 - 6x - 3 = 0$

....

(3)
(6 marks)

6. (a) Solve $x^2 + x + 11 = 14$

Give your solutions correct to 3 significant figures.

....

(3)

$y = x^2 + x + 11$ The value of y is a prime number when x = 0, 1, 2 and 3

The following statement is not true.

'$y = x^2 + x + 11$ is always a prime number when x is an integer'

(b)   Show that the statement is not true.

....

....

(1)

(4 marks)

Completing the Square

Things to remember:
- To complete the square:
    1. Halve the coefficient of x.
    2. Put in brackets with the x and square the brackets.
    3. Subtract the half-coefficient squared.
    4. Don't forget the constant on the end!
    5. Simply.
- For $(x - p)^2 + q = 0$, the turning point is (p, q).

Questions:
1.   (i)   Sketch the graph of $f(x) = x^2 - 5x + 10$, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

    (ii)   Hence, or otherwise, determine whether $f(x + 2) - 3 = 0$ has any real roots. Give reasons for your answer.

(Total for question = 6 marks)

2. (a) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

....
(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

....
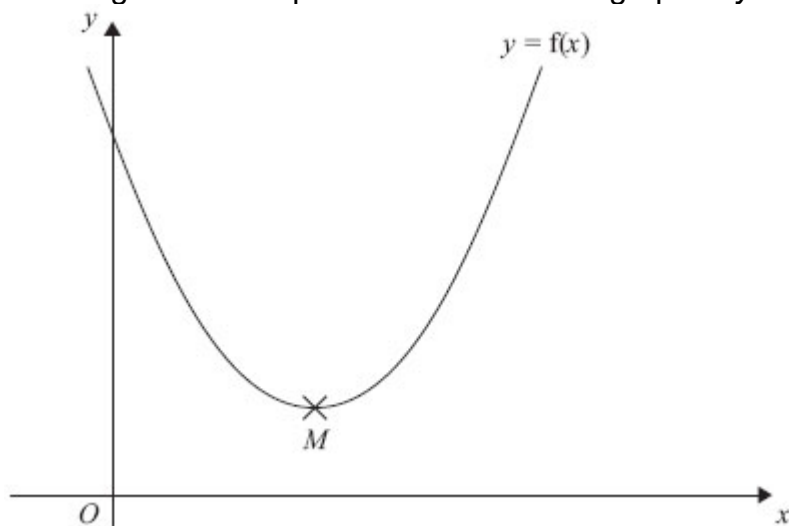**(1)**

(Total for question = 4 marks)

3. The expression $x^2 - 8x + 21$ can be written in the form $(x - a)^2 + b$ for all values of x.

(a) Find the value of a and the value of b.

a = ...

b = ...
(3)

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 8x + 21$
The diagram shows part of a sketch of the graph of $y = f(x)$.

The minimum point of the curve is M.
(b)   Write down the coordinates of M.

....
(1)
(Total for Question is 4 marks)

Algebraic fractions – solving

Things to remember:
- Multiply every term by the product of the denominators;
- Solve to find x.

Questions:

1. Solve $\dfrac{5(2x+1)}{3} = 4x + 7$

x = ...

(Total 3 marks)

2. (a) Show that the equation $\dfrac{5}{x+2} = \dfrac{4-3x}{x-1}$

can be rearranged to give $3x^2 + 7x - 13 = 0$

(3)

(b) Solve $3x^2 + 7x - 13 = 0$
Give your solutions correct to 2 decimal places.

x = ... or x = ...

(3)

(Total 6 marks)

3. Solve the equation $\dfrac{x}{2x-3} + \dfrac{4}{x+1} = 1$

$x =$ ....

4. Solve the equation $\dfrac{3}{x+3} - \dfrac{4}{x-3} = \dfrac{5x}{x^2-9}$

$x =$ ....

5. (a) Solve $\dfrac{3}{x} + \dfrac{3}{2x} = 2$

$x=$...

(2)

(b) Using your answer to part (a), or otherwise, solve $\dfrac{3}{(y-1)^2} + \dfrac{3}{2(y-1)^2} = 2$

$y = $.. or $y = $..

(3)

(Total 5 marks)

Nonlinear Simultaneous Equations

Things to remember:
1. Substitute the linear equation into the nonlinear equation.
2. Rearrange so it equals 0.
3. Factorise and solve for the first variable (remember there will be two solutions).
4. Substitute the first solutions to solve for the second variable.
5. Express the solution as a pair of coordinate where the graphs intersect.

Questions:
1. Solve the equations

$$x^2 + y^2 = 36$$
$$x = 2y + 6$$

....
(Total for Question is 5 marks)

2 Solve the simultaneous equations

$$x^2 + y^2 = 25$$
$$y = 2x + 5$$

x = ... and y = ...

or

x = ... and y = ...

(Total for Question is 6 marks)

3. Solve the simultaneous equations $x^2 + y^2 = 9$
$x + y = 2$

Give your answers correct to 2 decimal places.

$x =$ . . . . $y =$ . . . .

or $x =$ . . . . $y =$ . . . .

(Total for Question is 6 marks)

4. Solve algebraically the simultaneous equations
$$x^2 + y^2 = 25$$
$$y - 2x = 5$$